Optimization, Written Assignment #1

April 3, 2008

All numbered exercises are from Boyd and Vandenberghe.

1. Problem [2.4]
2. Problem [2.8]
3. Which of the following sets are convex? For sets that are not convex, give a counterexample to convexity.
   (a) The sets described in [2.12 (a)-(e)].
   (b) $\left\{x \in \mathbb{R}^n | \sum_{i=1}^n x_i^2 \leq 1,\, x_i \geq 0\right\}$.
   (c) $\left\{x \in \mathbb{R}^n | \sum_{i=1}^n x_i^2 = 1,\, x_i \geq 0\right\}$.
   (d) $\{x \in \mathbb{R}^n | \sum_{i=1}^n x_i \leq 1,\, x_i \geq 0\}$.
   (e) $\{x \in \mathbb{R}^n | \sum_{i=1}^n x_i = 1,\, x_i \geq 0\}$.
   (f) The set $\mathbf{S}^n$ of symmetric matrices in $\mathbb{R}^{n \times n}$.
   (g) The set of matrices of rank at most 1 in $\mathbb{R}^{n \times m}$.
   (h) The set of symmetric matrices with minimum eigenvalue at most 1.
   (i) Optional: [2.12 (f)-(g)]
   (j) Optional (but highly recommended at least to statistics and machine learning students): [2.15]
4. Problem [2.25] (Do draw the requested illustration, but there is no need to turn it in).
5. Show that if $f_1, \ldots, f_k$ are convex functions, then $f(x) = \max_i f_i(x)$ is also convex. Note that this holds also for supremum over an infinite set of functions.
6. We saw in class that every monotone increasing function on the reals $f : \mathbb{R} \to \mathbb{R}$ is quasiconvex (and in fact, quasi-linear). Give an example of a monotone increasing function that is not convex.
7. Problem [3.7]
8. Problem [3.16]
9. Optional (but highly recommended at least to statistics and machine learning students): [3.24]
10.
   (a) Problem [3.26 (a) and (b)]. $\mathbf{S}^n$ is the set of symmetric matrices in $\mathbb{R}^{n \times n}$. $\mathbf{S}_{++}^n$ is the set of symmetric positive definite matrices in $\mathbb{R}^{n \times n}$.
   (b) Show that the function from [3.26 (b)] is not concave over $\mathbf{S}^n$.